D. Operations (pp. 10–14)

Whole numbers, integers, and rational numbers are part of the set of real numbers. The following examples describe different operations with real numbers.

1. Find Opposites of Real Numbers

Opposite of a real number \(-a\) (read “the opposite of \(a\)”) is the same distance from 0 on a number line as \(a\), but it is on the opposite side of 0.

For the given value of \(a\), find \(-a\).

- \(a = 3\)
- \(a = \frac{7}{5}\)
- \(a = -5.4\)

Solution:

- \(-a = -(3)\)
- \(-a = -\left(\frac{7}{5}\right)\)
- \(-a = -(-5.4)\)

\(-a = -3\)
\(-a = -\frac{7}{5}\)
\(-a = 5.4\)

For the given value of the variable, find the opposite.

1. \(x = -6.2\)
2. \(u = 809\)
3. \(m = 0.25\)
4. \(w = \frac{45}{8}\)
5. \(k = -\frac{6}{11}\)
6. \(c = -8\frac{1}{2}\)

2. Find Absolute Values of Real Numbers

Absolute value of a real number \(|a|\) (read “the absolute value of \(a\)”) is the distance between \(a\) and 0 on a number line. If \(a\) is greater than or equal to 0, \(|a|\) is \(a\). If \(a\) is less than zero, \(|a|\) is the opposite of \(a\).

For the given value of \(a\), find \(|a|\).

- \(a = 8\)
- \(a = -\frac{4}{9}\)
- \(a = 11.5\)

Solution:

- \(|a| = |8| = 8\)
- \(|a| = \left|-\frac{4}{9}\right| = -\left(-\frac{4}{9}\right) = \frac{4}{9}\)
- \(|a| = |11.5| = 11.5\)

For the given value of the variable, find the absolute value.

7. \(b = 0.4\)
8. \(y = -50\)
9. \(p = -1.6\)
10. \(v = -9\)
11. \(n = \frac{23}{5}\)
12. \(h = 10\frac{8}{9}\)

3. Add Real Numbers

Sum The result of adding two or more real numbers.

To use a number line to find the sum of \(a + b\):

- Start at \(a\).
- If \(b > 0\), you will move to the right. If \(b < 0\), you will move to the left.
- Find \(|b|\) and move that many units.
- The number you stop on is the sum.
Use a number line to find the sum.

EXAMPLE

a. \( 4 + (-7) \)

Solution:
Start at 4. Move 7 units to the left.
End at -3.

\( 4 + (-7) = -3 \)

b. \(-2 + 3\)

Solution:
Start at -2. Move 3 units to the right.
End at 1.

\(-2 + 3 = 1\)

PRACTICE

Use a number line to find the sum.

13. \(-1 + (-4)\)

14. \(2 + (-8)\)

15. \(-5 + 9\)

16. \(-7 + 1\)

17. \(0 + (-2)\)

18. \(-3 + (-6)\)

Use the rules of real number addition to find the sum.

EXAMPLE

a. \(-19 + (-21)\)

Solution:
If two numbers have the same sign, add their absolute values. The sum has the same sign as the numbers added.

\(-19 + (-21) = -(|19| + |21|) = -(19 + 21) = -40\)

b. \(-\frac{1}{2} + \frac{3}{2}\)

Solution:
If two numbers have different signs, subtract the absolute value of the smaller number from the absolute value of the larger number. The sum has the same sign as the number with the larger absolute value.

\(-\frac{1}{2} + \frac{3}{2} = \frac{3}{2} - \left| -\frac{1}{2} \right| = \frac{3}{2} - \frac{1}{2} = 1\)

c. \(-2.8 + 1.5 = -(|-2.8| - |1.5|) = -(2.8 - 1.5) = -1.3\)

PRACTICE

Use the rules of real number addition to find the sum.

19. \(6.4 + (-0.3)\)

20. \(8 + (-10)\)

21. \(-100 + (-34)\)

22. \(s + 2\frac{2}{3} + 4\frac{3}{4}\)

23. \(-16 + 5\frac{1}{4}\)

24. \(55.1 + (-47.7)\)

4. Subtract Real Numbers

Difference The result of subtracting one real number from another real number.

Find the difference.

EXAMPLE

a. \(5 - 18\)

Solution:
To subtract \(b\) from \(a\), add \(a\) and the opposite of \(b\).

\(5 - 18 = 5 + (-18) = -13\)

b. \(-6 - 9\)

\(-6 - 9 = -6 + (-9) = -15\)

c. \(-8 - (-3)\)

\(-8 - (-3) = -8 + 3 = -5\)
Find the difference.

25. $-2.8 - 0.7$
26. $1.9 - 21.1$
27. $-34 - 57$
28. $\frac{3}{5} - (-4)$
29. $-\frac{24}{5} - \left(-\frac{16}{15}\right)$
30. $73 - (-82)$

5. Multiply Real Numbers

**Vocabulary**

Product The result of multiplying two or more real numbers.

**Example**

Find the product.

a. $-1.7(4)$

b. $-\frac{4}{5}(-10)$

c. $2(-3)(-8)$

Solution:

The product of two numbers with the same sign is positive and the product of two numbers with different signs is negative.

a. $-1.7(4) = -6.8$

b. $-\frac{4}{5}(-10) = \frac{40}{5} = 8$

c. $2(-3)(-8) = [2(-3)](-8) = -6(-8) = 48$

Find the product.

31. $10(-3)$
32. $-\frac{11}{4}(6)$
33. $\frac{-5}{8}\left(\frac{-24}{15}\right)\left(\frac{-25}{47}\right)$

34. $-12(4)(-3)$
35. $2.5(10.4)(-7)$
36. $-2.4(-9.1)$

**Vocabulary**

Multiplicative inverse of a real number $a$ The reciprocal of $a$, or $\frac{1}{a}$. The product of $a$ and its multiplicative inverse is 1.

**Example**

Find the multiplicative inverse of $a$.

a. $a = 9$

b. $a = -4$

c. $a = -\frac{1}{8}$

Solution:

a. $\frac{1}{a} = \frac{1}{9}$

b. $\frac{1}{a} = \frac{1}{-4} = -\frac{1}{4}$

c. $\frac{1}{a} = \frac{1}{-\frac{1}{8}} = -8 = -8$

Find the multiplicative inverse of the number.

37. $-6$
38. $1$
39. $\frac{3}{4}$

40. $-\frac{7}{5}$
41. $9\frac{1}{2}$
42. $-5\frac{15}{32}$
6. Divide Real Numbers

**Quotient** The result dividing a real number by another real number.

**Find the quotient.**

- **a.** $35 \div (-7)
  \[
  a = 35 \cdot \left( -\frac{1}{7} \right) = -\frac{35}{7} = -5
  \]
- **b.** $26 \div (-13)
  \[
  b = 26 \cdot \left( -\frac{1}{13} \right) = \frac{26}{13} = 2
  \]
- **c.** $\frac{2}{3} \div \left( -\frac{12}{18} \right)
  \[
  c = \frac{2}{3} \cdot \left( -\frac{18}{12} \right) = \frac{-36}{36} = -1
  \]

**PRACTICE** Find the quotient.

- **43.** $-92 \div (-4)
- **44.** $-2\frac{1}{4} \div \frac{5}{8}
- **45.** $9 \div \frac{1}{9}
- **46.** $1 \div \left( -\frac{5}{2} \right)
- **47.** $-\frac{32}{15} \div (-8)
- **48.** $-6\frac{2}{3} \div 10\frac{4}{9}$

7. Find Square Roots

**Square root of a** If $b^2 = a$, then $b$ is the square root of $a$. Every positive nonzero real number $a$ has two square roots, $-\sqrt{a}$ and $\sqrt{a}$.

**Radicand** The number or expression inside a radical symbol.

**Evaluate the expression.**

- **a.** $\pm \sqrt{49}
- **b.** $\sqrt{1}$
- **c.** $-\sqrt{144}$

**Solution:**

- **a.** $\pm 7
- **b.** $1$
- **c.** $-12$

**PRACTICE** Evaluate the expression.

- **49.** $-\sqrt{400}$
- **50.** $\pm \sqrt{9}$
- **51.** $\sqrt{81}$
- **52.** $\sqrt{0}$
- **53.** $\sqrt{4}$
- **54.** $\pm \sqrt{900}$
BENCHMARK 1
(Chapters 1 and 2)

Quiz
For the given value of the variable, find the opposite, absolute value, and multiplicative inverse.

1. \( a = -16 \)
2. \( y = 7 \frac{3}{10} \)
3. \( r = -0.3 \)

Evaluate the expression.
4. \( 51 - (-65) \)
5. \( -\frac{5}{7} \cdot \frac{21}{40} \)
6. \( \frac{5}{9} \div \left( -1 \frac{2}{3} \right) \)
7. \( 8 + (-15) \)
8. \( -2(-35) \)
9. \( \sqrt{64} \)
10. \( -18 \div \frac{12}{5} \)
11. \( \pm \sqrt{81} \)
12. \( -2.3 - 4.9 \)